5. V. P. Reutov, "Stability of the periodic deflection of a panel surface in a turbulent boundary layer," Prikl. Mekh. Tekh. Fiz., No. 4 (1992).
6. V. P. Reutov, "Stability of flexural vibrations of plates in a turbulent boundary layer," Prikl. Mekh. Tekh. Fiz., No. 1 (1993).
7. L. D. Landau and E. M. Lifshitz, Continuum Electrodynamics [in Russian], Nauka, Moscow (1982).
8. A. V. Gaponov-Grekhov, M. I. Rabinovich, and I. M. Starobinets, "Dynamic model of the spatial development of turbulence," Pisma Zh. Éksp. Teor. Fiz., 39, No. 12 (1984).
9. M. I. Rabinovich and M. M. Sushchik, "Regular and random dynamics of structures in fluid flow," Usp. Fiz. Nauk, 160, No. 1 (1990).
10. A. A. Andronov, A. A. Vitt, and S. É. Khaikin, Theory of Vibration [in Russian], 2nd edn., Fizmatgiz, Moscow (1959).
11. N. V. Butenin, Yu. I. Neimark, and N. A. Fufaev, Introduction to the Theory of Vibration [in Russian], Nauka, Moscow (1987).
12. N. N. Moiseev, Asymptotic Methods of Nonlinear Mechanics [in Russian], Nauka, Moscow (1969).
13. A. Lichtenberg and M. Lieberman, Regular and Random Dynamics [Russian translation], Nauka, Moscow (1984).
14. F. Moon, Random Vibration [Russian translation], Mir, Moscow (1990).
15. G. G. Schuster, Deterministic Chaos: Introduction [Russian translation], Mir, Moscow (1988).
16. J. Bekefy, Radiation Processes in Plasmas [Russian translation], Mir, Moscow (1971).
17. V. E. Zakharov, "Hamiltonian formalism for waves in nonlinear media with dispersion," Izv. Vyssh. Uchebn. Zaved., Radiofiz., 17, No. 4 (1974).

OPTIMUM WING SHAPES IN A HYPERSONIC NONEQUILIBRIUM FLOW

V. N. Golubkin and V. V. Negoda

UDC 533.6.011.55

The trajectories of aerospace vehicles include sections of hypersonic flight at an angle of attack characterized by substantial nonequilibrium flow over the bottom surface of the wing (lifting body) [1, 2]. The thin-shock-layer method has proven very fruitful for general study of the effect of nonequilibrium physicochemical processes on the flow field and the aerodynamic characteristics of wings. Using this approach for a gas of variable density, Stalker [3] generalized well-known solutions for a delta wing in the case of non-Newtonian flow. The ideas set forth in [4, 5] were used in [6, 7] to integrate a system of equations for a three-dimensional nonequilibrium shock layer on a low-aspect-ratio wing of arbitrary form. An effective method was proposed in [8] for solving the two-dimensional system of equations obtained by integration for the form of the surface of the lead shock. Proceeding on the basis of the analytical solution in [4], the authors of [9] formulated a variational problem involving determination of the configuration of a wing with optimum aerodynamic performance. Despite the fact that the flow field around the wing is three-dimensional — in contrast to the cases examined in [10] — the solution reduces to the minimization of a unidimensional functional. The results in [9] pertain to the limiting cases of a wing in a flow of an ideal gas or equilibrium reacting air.

In the present study, we propose a variational method of determining the shape of a wing which is to perform optimally under hypersonic conditions for the general case of chemically nonequilibrium flow. The solutions that are obtained reveal features of the design that allow an improvement in the aerodynamic performance of wings and lifting bodies in a relaxing hypersonic flow.

1. A highly approximate estimate for pressure is obtained from the limiting Newtonian scheme of hypersonic flow with an infinitesimally thin shock layer on the surface of a body and the density ratio on the coincident lead shock $\rho_\infty/\rho_s^0 = \varepsilon = 0$. Examining the next approximation, with small nontrivial values of ε, makes it possible to more accurately evaluate

Zhukovskii. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 2, pp. 66-72, March-April, 1993. Original article submitted February 7, 1992.

 0021-8944/93/3402-0212$12.50

the flow characteristics by studying the structure of the flow in the shock layer. The latter is affected by the character of the physicochemical processes which occur in the gas at high temperatures, i.e., by whether these processes are frozen, equilibrium, or intermediate (nonequilibrium) [1, 2]. In both of these limiting cases, flow over the windward side of a thin low-aspect wing is in principle analogous to the flow of an ideal gas with the adiabatic exponent $\kappa \to 1$ [4, 11, 12].

We will examine a substantially nonequilibrium flow, assuming that the order of magnitude of the extension of the wing is the same as that of the Mach angle in the shock layer ($\varepsilon^{1/2}$ tan α) and that the relative thickness is of the same order as the thickness of the layer (ε tan α). With small ε and finite angles of attack α, the main sought functions can be represented in the form of expansions

$$\begin{gathered} u^0/V_\infty = \cos\alpha + \varepsilon u \sin\alpha \operatorname{tg}\alpha + \ldots, \\ v^0/V_\infty = \varepsilon v \sin\alpha + \ldots, \quad w^0/V_\infty = \varepsilon^{1/2} w \sin\alpha + \ldots, \\ (p^0 - p_\infty)/(\rho_\infty V_\infty^2) = (1 + \varepsilon p)\sin^2\alpha + \ldots, \\ \rho^0/\rho_\infty = \varepsilon^{-1}\rho + \rho_1 + \ldots, \quad 2h^0/V_\infty^2 = h \sin^2\alpha + \ldots, \end{gathered} \tag{1.1}$$

where u^0, v^0, and w^0 are the components of the velocity vector in the coordinate system $x^0y^0z^0$ connected with the wing; p^0, ρ^0, and h^0 are pressure, density, and enthalpy; and V_∞, p_∞, and ρ_∞ are the parameters of the incident flow. The functions u, v, w, and p, dependent on the dimensionless coordinates (where L is the length of the wing)

$$x = x^0/L, \quad y = y^0/\varepsilon L \operatorname{tg}\alpha, \quad z = z^0/\varepsilon^{1/2} L \operatorname{tg}\alpha, \tag{1.2}$$

are determined from the "gas dynamic" part of the complete system of equations in the given approximation

$$\begin{gathered} Du = 0, \quad Dv = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \quad Dw = 0, \\ D\ln\rho + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad D \equiv \frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} \end{gathered} \tag{1.3}$$

Here, the boundary conditions on the lead shock $y = S(x, z)$ and on the wing $y = B(x, z)$ are as follows

$$\begin{gathered} u_s = -\frac{\partial S}{\partial x}, \quad v_s = \frac{\partial S}{\partial x} - \left(\frac{\partial S}{\partial z}\right)^2 - 1, \quad w_s = -\frac{\partial S}{\partial z}, \\ p_s = -1 - w_s^2 - 2u_s, \quad \rho_s = 1, \quad y = S(x, z), \quad v = \frac{\partial B}{\partial x} + w\frac{\partial B}{\partial z}, \quad y = B(x, z). \end{gathered} \tag{1.4}$$

It turns out that the ratio of the continuous component of vorticity to the density of the gas remains constant along the streamlines of the given three-dimensional flow [5]

$$D\left(\frac{1}{\rho}\frac{\partial w}{\partial y}\right) = 0. \tag{1.5}$$

With an arbitrary function ρ, this allows us to integrate nonlinear three-dimensional system (1.3) and, with allowance for boundary conditions (1.4), reduce it to two-dimensional equations relative to the form of the lead shock:

$$S(x, z) = B(x, z) + \int\limits_{\xi_b(x,z)}^{x} \frac{\partial \zeta}{\partial z}(x, \xi, z)\frac{d\xi}{\rho(x - \xi)}; \tag{1.6}$$

$$\zeta = z + (x - \xi)\frac{\partial S}{\partial z}(\xi, \zeta). \tag{1.7}$$

Here, $x = \xi$, $z = \zeta$ are the coordinates of the point of entry of a streamline into the shock layer; ξ_b pertains to the surface streamlines. Finding the main function S(x, z), we express all of the remaining sought functions in the form of integral and functional relations. For example,

$$u = -\frac{\partial S}{\partial x}(\xi, \zeta), \quad y = B(x, z) + \int_{\xi_b}^{\xi} \frac{\partial \zeta}{\partial z}(x, \xi', z) \frac{d\xi'}{\rho(x - \xi')}. \tag{1.8}$$

In turn, the still-arbitrary function ρ is determined from the second, "chemical-kinetics" part of the problem. In contrast to three-dimensional system (1.3), it is a set of unidimensional enthalpy equations containing only derivatives along the streamlines:

$$Dh = \left.\frac{\partial h}{\partial x}\right|_{\xi,\zeta} = 0, \quad h_s = \text{const}, \tag{1.9}$$

The function also contains a set of parameters q_n [2] characterizing the composition and state of the gas mixture in the shock layer with constant pressure (1.1) and enthalpy (1.9) in the principal term. Thus, the functional form of the latter and the equation of state of density determined in terms of these quantities will be $f(x - \xi)$. This expression has already been used in (1.6), (1.8). Since the streamlines around a low-aspect wing are nearly rectilinear in the given analytical approximation, density $\rho(x - \xi)$ can be found [3, 6] by numerically calculating the unidimensional flow of the relaxing air behind the shock wave. Analysis of the numerical results in [1] showed that in the region in which the law of binary simulitude is valid (height $H \gtrsim 40$ km) and with velocities on the order of the escape velocity, we can approximate ρ in the form

$$\rho(x - \xi) = \begin{cases} 1, & 0 \leqslant x - \xi \leqslant \sigma_f, \\ 1 + K_n \ln \dfrac{x - \xi}{\sigma_f}, & \sigma_f \leqslant x - \xi \leqslant \sigma_{eq}, \\ 1 + \Delta\rho, & x - \xi \geqslant \sigma_{eq}, \end{cases} \tag{1.10}$$

where $K_n = \Delta\rho / \ln(\sigma_{eq}/\sigma_f)$; σ_f, σ_{eq} are the dimensionless lengths of the region of frozen flow near the shock and the relaxation length (usually, $\sigma_f \ll \sigma_{eq}$); and $\Delta\rho$ is the difference between the equilibrium and frozen values of density. The quantities σ_f and σ_{eq} depend on the binary similitude parameter $\rho_\infty L$ and flight velocity. The flow around the wing can be considered frozen at $\sigma_f > 1$ and equilibrium at $\sigma_f < \sigma_{eq} \ll 1$. In both cases, ε is chosen so that $\rho \equiv 1$.

It is interesting that, in accordance with (1.5), the increase in density which occurs during relaxation (1.10) also results in an increase in vorticity downstream.

2. If we use the integral momentum theorem, we find that the normal and axial forces acting on the bottom surface of a wing with a shock attached to its leading edge can be expressed through the integrals over the aft section of the shock layer Σ_c in the plane $x = 1$.

$$\frac{N}{\rho_\infty V_\infty^2 L^2} = \varepsilon^{1/2} \sigma (1 + \varepsilon P) \sin^2\alpha \operatorname{tg} \alpha + \ldots,$$

$$\frac{T}{\rho_\infty V_\infty^2 L^2} = -\varepsilon^{3/2} \sigma \left(\frac{\sigma_c}{\sigma} - R\right) \sin^2\alpha \operatorname{tg}^2\alpha + \ldots,$$

$$P = \frac{\sigma_c}{\sigma \varkappa_\infty \varepsilon M_\infty^2 \sin^2\alpha} + \frac{1}{\sigma} \iint_{\Sigma_c} [\rho_1 + \rho(v + u \operatorname{tg}^2\alpha)]\, dy\, dz,$$

$$R = -\frac{1}{\sigma} \iint_{\Sigma_c} \rho u\, dy\, dz, \quad \sigma = \iint_{\Sigma} dx\, dz, \quad \sigma_c = \iint_{\Sigma_c} dy\, dz.$$

The region Σ is bounded by the projection of the leading edge on the plane $y = 0$ $|z| = z_e(x)$ or $x = x_e(z)$ and the trailing edge $x = 1$, while the region Σ_c is bounded by the lines $B(1, z)$ and $S(1, z)$. Changing over to the continuous coordinate system, we obtain the following for aerodynamic performance in the given non-Newtonian approximation

$$K = \operatorname{ctg} \alpha - \frac{2\varepsilon}{\sin 2\alpha} Q, \quad Q = R - \frac{\sigma_c}{\sigma}. \tag{2.1}$$

It is evident from this that, in principle, the thin-shock-layer method is better than the approach based on the Newtonian limit for a place in regard to the accuracy of determination of the gasdynamic functions.

TABLE 1

σ	z_0	x_1	A	Q_{min}
2,6	0	1	—0,48	—1,12
3	0,07	1	—0,37	—1,16
3,2	0,175	1	—0,3	—1,17

It is very important that the double integrals in (2.1) are transformed into single integrals when analytic solution (1.6-1.8) is used. In fact, if we follow [9] and integrate over ξ and ζ, we obtain the following in the case of variable density

$$\sigma = \int_{-b}^{b} [1 - x_e(z)]\, dz, \quad \sigma_c = 2\int_{0}^{1} z_e(x)/\rho\,(1-x)\, dx,$$
$$R = \frac{1}{\sigma}\int_{-b}^{b} \{S(1, z) - S[x_e(z), z]\}\, dz, \quad b = z_e(1). \tag{2.2}$$

As a result, the problem of optimizing wing shape to maximize aerodynamic performance at hypersonic speeds reduces to finding the minimum of the unidimensional functional Q (2.1). Here, we assume that the flight regime (V_∞, H, α) and the length of the wing L are given and that we assign the area of the wing in plan σ (i.e., buoyancy) as an isoperimetric condition.

It is evident from (2.1-2.2) that $R = \sigma_c/\sigma$ and $Q = 0$ for a flat wing ($B = 0$) with an edge of arbitrary shape in plan (assuming that the shock is attached). Fairly large gains in performance compared to cot α in the Newtonian limit or the performance for a flat wing can be obtained by choosing the shape of the wing in plan and the transverse strains of the wing's surface. As a result, $R < 0$, i.e., the projection of the leading edge on the plane $x = 1$ lies below the aft section of the shock. The corresponding optimum form of the shock will be sought in the class of functions

$$S(x, z) = kz^n \ln(\delta + x), \tag{2.3}$$

where k, n, and δ are arbitrary form factors. With a constant density ($\rho = 1$), the area of the wing in plan will be equal to the area of the aft section of the shock layer $\sigma = \sigma_c$. Meanwhile, the functional R will have the same form for both variable density and for $\rho = 1$. In this case, the main functions entering into R (or Q) are the form of the shock $y = S(x, z)$ and the projections of the leading edge. After they are found, the form of the surface of the wing is determined from (1.6-1.7), (1.10) using the method in [9]. The existence of the solution of this inverse problem is one more condition to be met in the solution of the variational problem. The latter is divided into two parts. The first such condition is optimizing the contour of the wing in plan while the form of the shock is given. Resulution of this problem in a manner similar to [9] yields $R_{min} > 0$ at $k > 0$ for a wing with a bow edge $z_e(0) \neq 0$. An even greater increase in performance is obtained at $R < 0$ ($k < 0$), although here we have a maximum $R_{max} < 0$ rather than a minimum. Since K_{min} is then greater than K_{max} for $k > 0$, the solution of the second part of the variational problem on optimizing the form of the shock (the parameters k, n, and δ) should be sought in the region $k < 0$. If this is the case, then performance $K > K_{min}$ for any wing shape different from that obtained in the first stage [9]. However, fairly severe restrictions are imposed on it by the above condition regarding the existence of the solution of the inverse problem (which unfortunately is not stated in analytical form). We therefore solved the optimization problem by a direct numerical method with allowance for the tendencies demonstrated in [9] in regard to the effect of the form factors. Specifically, we selected the most suitable factors and checked for satisfaction of the given condition numerically. To eliminate physically unrealistic edge shapes and those that violate this condition, we also assigned the span of the wing 2b and required that $z'_e(x) \geq 0$ on most of it. All of the characteristic configurations are covered by the function $z_e(x)$ in the form of a cubic parabola with respect to an argument reckoned from the point $z_0 = z_e(0) \geq 0$ along an axis making an angle θ with the x axis:

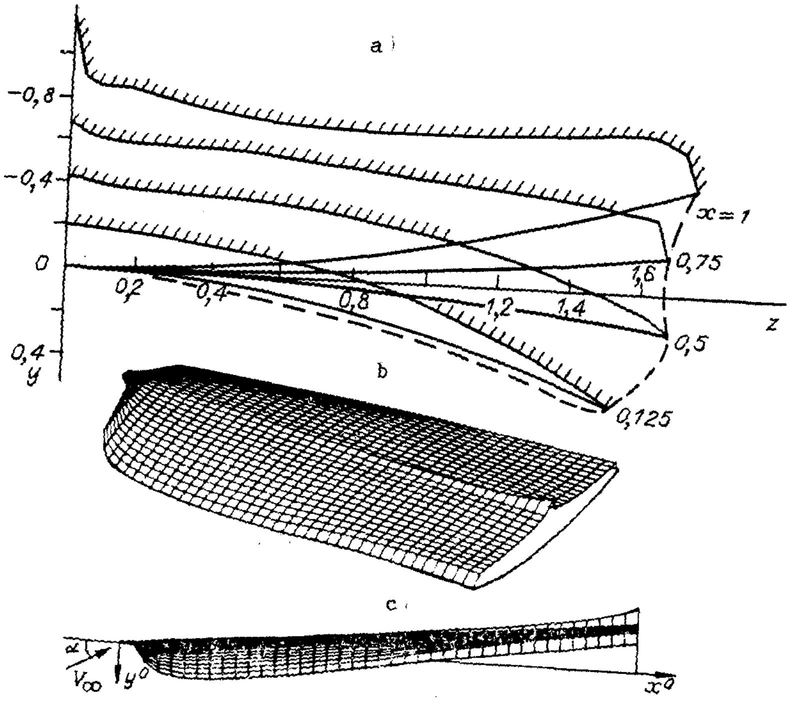

Fig. 1

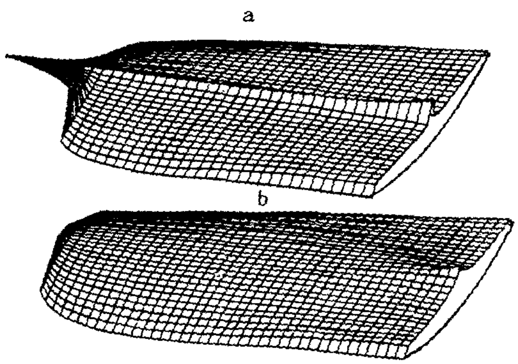

Fig. 2

$$
\begin{gathered}
z_e(x) = \begin{cases} z_0 + x \operatorname{tg}\theta + \Delta z_e(x), & 0 \leqslant x \leqslant x_1, \\ b, & x_1 \leqslant x \leqslant 1, \end{cases} \\
\Delta z_e \cos\theta = F(x/\cos\theta + \Delta z_e \sin\theta), \quad x_1 \operatorname{tg}\theta = b - z_0, \\
F(\eta) = A\eta(\eta - l_1)(\eta - l_2), \quad l_2^2 = x_1^2 + (b - z_0)^2.
\end{gathered} \tag{2.4}
$$

Here, the parameter ℓ_1 is determined from the assigned area σ:

$$
l_1 = \frac{l_2}{2} + \frac{6\sigma_1}{A l_2^3}, \quad \sigma_1 = \frac{1}{2}[\sigma - b(2 - x_1) - z_0 x_1];
$$

Along with k, n, and δ, the parameters z_0, x_1, and A were varied within the limits

$$
0 \leqslant z_0 \leqslant b,\ 0{,}5 \leqslant x_1 \leqslant 1,\ -1 \leqslant A \leqslant 1,\ -0{,}5 \leqslant k \leqslant 0,\ 2 \leqslant n \leqslant 3,\ 0 \leqslant \delta \leqslant 0{,}5,
$$

which ensured that we would consider a wide range of configurations — since the root ℓ_2 and the point of inflection $\eta_* = (\ell_1 + \ell_2)/3$ could be located either within the interval $[0, \ell_2]$ or outside it. In general, the given approach is similar to that used in [13]. In the analysis of the functional Q, for convenience the variable part of density in (1.10) was represented in the equivalent form $[(x-\xi)/\sigma_j]^{K_n}$ [8], since $K_n \ll 1$.

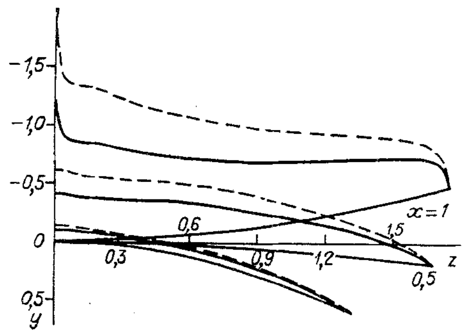

Fig. 3

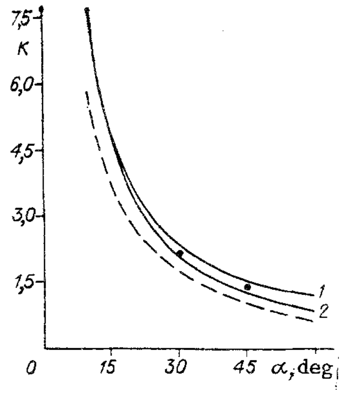

Fig. 4

3. Let us present some results of using the variational method developed here. For the case when the trajectory parameters are $M_\infty = 20$, $H = 60$ km, and $\alpha = 30°$ ($\varepsilon = 0.175$) and the parameters of the nonequilibrium density distribution $\sigma_f = 10^{-3}$, $\sigma_{eq} = 1$, $K_n = 0.07$ with assigned dimensionless values for the area in plan $\sigma = 3$ and the span $2b = 3.5$, we obtained the form factors for the optimum wing in (2.3-2.4) $k = -0.5$, $n = 2$, $\delta = 0.38$, $z_0 = 0.07$, $x_1 = 1$, $A = -0.37$ and the value $Q_{min} = -1.16$. In terms of aerodynamic performance, this gives $K_{max} = 2.2$. Figure 1a uses corrected coordinates (1.2) to show cross sections of the surface of this wing (the hatched lines) and the lead shock (solid lines) in the planes x = const (the dashed line shows the projection of the leading edge on the plane $x = 1$). Figure 1, parts b and c use the initial coordinates to show the overall shape of the bottom surface of the wing and the lateral view. The gain in performance compared to the Newtonian value $K_N = 1.73$ is realized due to the fact that the part of the surface adjacent to the leading edge where the width of the wing increases abruptly is bent downward. As a result, the projection of the leading edge on the aft surface $x = 1$ lies below the section of the shock coincident with this plane. In this case, the functional $R < 0$ (2.2). The optimum wing has an inwardly concave bottom surface and a characteristic central depression in the aft part. The presence of the depression is probably connected with the optimum properties of star-shaped bodies [14], these same properties being intrinsic to configurations of the given class. A change in the area σ affects mainly the parameters of the shape in plan (see Table 1). Meanwhile, a decrease in σ is accompanied by a decrease in the gain in performance. If the assigned area σ is close to the area of the circumscribed rectangle $2b \times 1$, then the optimum wing has a bow edge (Fig. 1b, Fig. 2b). This shape disappears with a decrease in σ (Fig. 2a).

Comparison of the results of optimization with the same trajectory parameters in a flow of an ideal gas ($\kappa = 1.4$) showed that nonequilibrium has almost no effect on the form of the optimum wing in plan but does change the configuration of the bottom surface. The latter change occurs because the thickness of the shock layer decreases as a result of a relaxational increase in density. There is also a decrease in K_{max} (at $\sigma = 3$, $K_{max} = 2.2$ instead of $K_{max} = 2.34$ for $\kappa = 1.4$). The dashed lines in Fig. 3 show cross sections of the wing at $\kappa = 1.4$, while the solid hatched lines show cross sections in the nonequilibrium flow. If instead of $\sigma_{eq} = 1$ we take $\sigma_{eq} = 0.2$ — which is equivalent to a fivefold increase in the length of the wing and more advanced relaxation — then instead of $Q_{min} = -1.16$ we obtain $Q_{min} = -1.13$. Accordingly, $K_{max} = 2.2$ and 2.19. In the equilibrium limit, we have $\varepsilon_{eq} = 0.093$, $K_{max} = 2.05$.

Figure 4 shows the effect of the actual properties of air on the function $K_{max}(\alpha)$ ($M_\infty = 22$, $H = 70$ km). Here, line 1 corresponds to $\kappa = 1.4$, while line 2 corresponds to flow of equilibrium reacting air. The points depict nonequilibrium flow about a wing of length $L = 14$ m with angles of attack $\alpha = 30$ and 45°. Despite the fact that the effect of the real gas is to reduce K_{max}, its value nonetheless remains greater than in the Newtonian limit $K_N = \cot\alpha$ (the dashed line in Fig. 4).

Allowance for viscosity with the assumption of a constant friction coefficient showed that it has almost no effect on the configuration of the optimum wing. However, in this case, the decrease in maximum performance is greater, the smaller the angle of attack.

In using parametric relation (2.3) to specify the functional form of the shock, we set out mainly to determine the geometric features of the wing that might improve performance. As regards the value of K_{max}, it may be somewhat greater for wings having different features generating a shock functionally different from (2.3).

In conclusion, we note that the optimum shape obtained for the bottom surface of the wing is also the optimum shape for a bottom surface of a lifting body. The top surface of the latter can be formed (for example) to conform to the streamlines of the undisturbed flow passing over the leading edge. For such a body, the exchange coefficient $\tau = V/S^{3/2} \sim 1$.

The authors thank G. I. Maikapar for supporting our efforts in this study.

LITERATURE CITED

1. G. I. Maikapar (ed.), Nonequilibrium Physicochemical Processes in Aerodynamics [in Russian], Mashinostroenie, Moscow (1972).
2. V. V. Lunev, Hypersonic Aerodynamics [in Russian], Mashinostroenie, Moscow (1975).
3. R. J. Stalker, "Nonequilibrium flow over delta wings with detached shock waves," AIAA J., 20, No. 12 (1982).
4. A. I. Golubinskii and V. N. Golubkin, "Three-dimensional hypersonic flow of a gas about a thin wing," Dokl. Akad. Nauk SSSR, 234, No. 5 (1977).
5. A. I. Golubinskii and V. N. Golubkin, "Three-dimensional hypersonic flow about a body of finite thickness," Uch. Zap. TsAGI, 13, No. 2 (1982).
6. A. I. Golubinskii and V. N. Golubkin, "Hypersonic flow of a nonequilibrium gas about a low-aspect-ratio wing," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1983).
7. M. M. Kuznetsov, "Theory of hypersonic three-dimensional nonsteady flow of a relaxing gas about a wing of arbitrary aspect ratio," Prikl. Mekh. Tekh. Fiz., No. 5 (1983).
8. V. N. Golubkin and V. V. Negoda, "Numerical calculation of nonequilibrium flow about a wing in a thin-shock-layer approximation," Zh. Vychisl. Mat. Mat. Fiz., 25, No. 4 (1985).
9. V. N. Golubkin and V. V. Negoda, "Optimization of the three-dimensional form of low-aspect ratio lifting bodies at hypersonic velocities," ibid., 31, No. 12 (1991).
10. V. A. Shchepanovskii, Gas dynamic Design [in Russian], Nauka, Novosibirsk (1991).
11. V. N. Golubkin, "Hypersonic flow of a gas about a flat triangular wing," Uch. Zap. TsAGI, 7, No. 6 (1976).
12. V. N. Golubkin and V. V. Negoda, "Calculation of the flow around delta wings in a thin-shock-layer approximation," Zh. Vychisl. Mat. Mat. Fiz., 29, No. 10 (1989).
13. Yu. A. Vedernikov, V. G. Dulov, and A. F. Latypov, "Optimization of hypersonic three-dimensional configurations," Prikl. Mekh. Tekh. Fiz., No. 1 (1979).
14. A. L. Gonor, "Three-dimensional bodies with minimum drag at high supersonic velocities," Prikl. Mat. Mekh., 27, No. 1 (1963).